

$$A \times (B \times C) = (A \cdot C)B - (A \cdot B)C \neq (A \times B) \times C,$$

$$\text{Lagrange' identity: } (v \times w) \cdot (a \times b) = (v \cdot a)(w \cdot b) - (v \cdot b)(w \cdot a)$$

$$\frac{ds}{dt} = |\alpha'(t)| = \nu(t), \quad T(t) := \frac{\alpha'(t)}{\nu(t)}, \quad \kappa(s) := \left| \frac{dT}{ds} \right|, \quad N(s) := \frac{\frac{dT}{ds}}{\kappa(s)}, \quad B := T \times N, \quad \tau(s) := \frac{dN}{ds} \cdot B$$

$$\begin{aligned} \frac{dT}{dt} &= \frac{dT}{ds} \frac{ds}{dt} = \kappa \frac{ds}{dt} N \\ \frac{dN}{dt} &= \frac{dN}{ds} \frac{ds}{dt} = -\kappa \frac{ds}{dt} T + \tau \frac{ds}{dt} B \\ \frac{dB}{dt} &= \frac{dB}{ds} \frac{ds}{dt} = -\tau \frac{ds}{dt} N. \end{aligned}$$

$$\text{The normal curvature in the direction of a unit vector } u: \quad k(u) := S_p(u) \cdot u$$

$$\text{Euler's formula: } \quad k(u) = k_1 \cos^2 \theta + k_2 \sin^2 \theta$$

$$S(\varphi_u) \cdot \varphi_u = \varphi_{uu} \cdot U, \quad S(\varphi_u) \cdot \varphi_v = S(\varphi_v) \cdot \varphi_u = \varphi_{uv} \cdot U, \quad S(\varphi_v) \cdot \varphi_v = \varphi_{vv} \cdot U$$

$$\begin{pmatrix} l & m \\ m & n \end{pmatrix} = \begin{pmatrix} E & F \\ F & G \end{pmatrix} \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$K = \frac{ln - m^2}{EG - F^2}, \quad H = \frac{Gl + En - 2Fm}{2(EG - F^2)}, \quad k_1 = H + \sqrt{H^2 - K}, \quad k_2 = H - \sqrt{H^2 - K}$$

$$\kappa_\alpha^2 = k_n(T_\alpha)^2 + (k_g)_\alpha^2$$

$$\text{The geodesic equations in the case } F = 0: \quad u'' + \frac{E_u}{2E} u'^2 + \frac{E_v}{E} u'v' - \frac{G_u}{2E} v'^2 = 0, \quad v'' + \frac{G_v}{2G} v'^2 + \frac{G_u}{G} u'v' - \frac{E_v}{2G} u'^2 = 0.$$

$$\begin{aligned} \nabla_v E_1 &= \omega_{12}(v)E_2 + \omega_{13}(v)E_3 \\ \nabla_v E_2 &= \omega_{21}(v)E_1 + \omega_{23}(v)E_3 \\ \nabla_v E_3 &= \omega_{31}(v)E_1 + \omega_{32}(v)E_2 \end{aligned}$$

In \mathbb{R}^3 :

$$\begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = A \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}, \quad \Theta = A \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}, \quad \Omega = (dA)(A^t), \quad \text{Structural equations: } d\Theta = \Omega \wedge \Theta, \quad d\Omega = \Omega \wedge \Omega$$

On a surface:

$$\text{First structural equations: } \quad d\theta_1 = \omega_{12} \wedge \theta_2, \quad d\theta_2 = \omega_{21} \wedge \theta_1$$

$$\text{Symmetry equation: } \quad \omega_{31} \wedge \theta_1 + \omega_{32} \wedge \theta_2 = 0$$

$$\text{Gauss equation: } \quad d\omega_{12} = \omega_{13} \wedge \omega_{32}$$

$$\text{Codazzi equations: } \quad d\omega_{13} = \omega_{12} \wedge \omega_{23}, \quad d\omega_{23} = \omega_{21} \wedge \omega_{13}$$

$$d\omega_{12} = \omega_{13} \wedge \omega_{32} = -K\theta_1 \wedge \theta_2, \quad \omega_{13} \wedge \theta_2 + \theta_1 \wedge \omega_{23} = 2H\theta_1 \wedge \theta_2$$

$$\text{Matrix of } S \text{ on the basis } \{E_1, E_2\}: \quad \begin{pmatrix} \omega_{13}(E_1) & \omega_{13}(E_2) \\ \omega_{23}(E_1) & \omega_{23}(E_2) \end{pmatrix}$$